
Lively Math Teaching— What Is It? How Is It Done?

By Harley Paul Larkin

Careful observation of role models in math teaching and experience from years of teaching have taught me that certain elements must be present before one can expect to teach an exciting and profitable mathematics class.

First and most important, as a teacher you must make God your constant partner and daily partake of the bread of life. Mathematics, as we know it, is man's discovery of the rules by which God's orderly universe is run.

In order to be effective, the mathematics teacher needs a broad range of knowledge in his or her field, as well as an enthusiasm about sharing it with students. It is impossible to share what you do not possess; you cannot inspire enthusiasm about your subject in others unless you feel it yourself.

Be flexible. Don't be afraid to try a new approach—innovation can lead to creative, exciting teaching. Attend in-service conferences and continue your education so that you'll stay up to date. Nobody wants to sail with the mothball fleet.

Set rules and guidelines for yourself and stay by them until a

better approach can be substituted. Major switches, however, should be made at the beginning of the semester, as midstream changes can drown the student.

View yourself realistically and maintain a sense of humor. Mathematics is an exact science, but this does not necessarily make you exact or infallible. Admit it when you are stumped. Tell the students, "I am unable to come up with the solution at this time. I will give it some study, and get back to you tomorrow." Even famous mathematicians make errors and at times must lay aside their work to think. Allow the same parameters

for yourself and your students.

Now we come to the actual business of conducting an effective mathematics class. First of all, before a single student enters your classroom, you must compile concise yearly goals for your pupils. Ask yourself, What do I want them to have accomplished at the end of this school year? Have I included sufficient topics to prepare them for their next educational step? Will this class dovetail with advanced classes the student will take later as he or she attempts to develop marketable skills? Does the classwork fulfill state and denominational requirements?

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Strong Teacher—Strong Textbook

Next, you should carefully inspect your textbook, selecting supplementary resources to strengthen weak areas. Since you have clearly defined the class's learning objectives and goals, the weak areas will be easier to pinpoint.

Consider the following illustration:

Beginning with the primary grades, the teacher should recognize and understand how real number properties relate to the level he or she is teaching. Two examples are these: (1) *The Order Property*, e.g., mathematical operations are done from left to right on a number sentence with the taxonomy of powers, first multiplication and division, and lastly, addition and subtraction. (2) *Real Number Postulates*, e.g., commutative, associative, identity, inverse, distributive, and closure.

To a teacher not thoroughly versed in real number properties, the following is just a plain old multiplication problem that students must somehow be taught the proper steps to solve.

$$\begin{array}{r} 235 \\ \times 14 \\ \hline 940 \\ 2350 \\ \hline 3290 \end{array}$$

To the mind of the prepared teacher this problem can be thought of in terms of the Distributive Property of Multiplication over Addition as follows: $(4 + 10)(235) = (4 \times 235) + (10 \times 235) = 940 + 2350 = 3290$.

If inspection shows that your mathematics textbook is weak in too many basic skills areas, change books. Usually you will find several different books approved by both state and conference to choose from. If your school budget or administration does not allow a change of texts, the market

is flooded with workbooks and computer software. Review such materials carefully, however, to be sure that they reinforce your instructional objectives.

Making a Big Map

After a great deal of experimentation, I have discovered that I achieve the best results when I map out the whole year in advance. When this is not done, some foundation areas may be minimized or neglected. Such mapping also allows me to decide in advance how much time will be spent in each unit and how to best use my time and that of my students.

An important part of mapping out the year is to set unit goals. A goal must first be established before it can be achieved, just as a postulate must be learned before it can be applied. My weekly and daily schedules help keep me on course and enable me to quickly review long-term objectives in terms of where I have been. This mapping also helps me cue my students about the direction this concept is leading them.

Don't Forget to Read the Map

Every day's class should include two steps: (1) A look backward, and (2) a look forward to where we are heading in today's lesson. Unit and chapter overviews with the class become a third step that gives meaning to the student. Such overviews help the student to understand the course's broad objectives, as well as its short-term goals. If the text does not provide for systematic review, schedule it yourself. Everyone's mind (including the teacher's!) needs updating regularly.

Creating Internal Excitement

Begin the year by saying, "I know that there will be times when

things will seem hard for you; however, I believe that you are going to find this a very exciting course. I predict that all of you will do much better than you ever did before in math." Then keep up the students' internal excitement with the constant positive affirmation: "You can do it. I believe in you." Students must be driven from within to achieve consistently superior performance.

Leave Tracks That Are Easy to Follow

One key ingredient of a successful math experience for the student and the teacher is this: You as teacher must insist on clearly headed, well organized, neatly written papers. This requirement will give the student pride in his or her work and make it easy for you to correct the paper and trace any errors that have crept in.

The final solution of a problem is no more important than where the solution originated. All steps, from copying the original problem through the steps of solving it to the final solution should be required—written clearly and in logical order. This allows both teacher and student to readily trace all components and shows whether the student understands the steps involved in arriving at the answer, even if he or she has made calculation errors along the way. Even if the student has received help from others, having to write out the complete process of solving the problem will offer insights into problem solving.

See, I Can't Grasp What I Hear You Saying

Since each student can be categorized primarily as either a visual, aural, or tactile learner, and no class is made up of only one of these types, the teacher should carefully frame questions and re-

sponses and provide examples and models that relate equally to each learning style. Teachers need to be alert in this regard, or students will receive most of their information based on the *teacher's* dominant characteristic.

When questions such as "Why do I need to study this?" arise, every teacher should be ready to give an answer that the student can understand. A well-prepared teacher has researched the reasons why the text is written as it is and can add as many practical applications as are needed to satisfy the student.

Too often, when asked such questions as "Why not divide by zero?" the teacher answers simply "Because I say so" or "Because it is a rule, and you must follow the rules." This type of answer frequently is heard when the teacher's knowledge is limited to the materials in the textbook. An even worse indictment of this method of teaching is its failure to train the student to figure out for himself the reason for a rule or a prescribed technique in problem solving.

A better answer to the question about dividing by zero would be this: "Consider the following problem: $4 \div 0 = a$. This is equivalent to $4 = a \times 0$. Can anyone tell me what real number will substitute for a to make the sentence true?" It will soon become apparent that no such number exists and hence the sentence can never be accurate if division by zero occurs.

Feed Me, I'm Starving for Information

The "carrot or the stick" philosophy does have some merit. If a human being does not receive prompt and appropriate feedback, he or she tends to lose interest. One of my continuing priorities is for my students to always receive their

graded work by the following school day. This serves three purposes. First, I know which areas to compliment; second, if there is a problem, we can correct it together before it becomes entrenched; and third, the students feel confident about themselves as their successes mount daily. This helps lessen the impact of a temporary setback.

If I were asked, "What has been most helpful to you as a teacher and to your classes?" my answer would be divided into three parts: "I make available to all students their cumulative grade (updated weekly by my computer helper). Students receive a listing of the assignments a quarter at a time (I am now experimenting with semester and full-year blocks), and I always pray with the students, especially before testing."

Common Sense Rules

My rules in the classroom are few, to the point, and enforceable: (1) No late work will be accepted. Occasionally, common sense dictates that I make exceptions, but the rule remains intact. (2) I set deadlines and make them public. It pays to advertise. (3) If I err in my favor, I will change the score or grade. If I err in the student's favor, he keeps the points. This rule has never biased anyone's grade. On the contrary, it encourages students to examine their returned work before disposing of it and makes them even more determined to earn the grade I know they are capable of achieving. Teachers must be perceived as fair, or they will not be able to effect positive changes in their students' behavior and attitudes.

ZZZZZ . . .

I have discovered that math lectures that drag on and on are seldom heard, little loved, and rarely remembered. I now keep my com-

ments to the class as a whole to a minimum and maximize my time with one-on-one instruction. This allows me to encourage the slow learner who will not, for fear of embarrassment, ask questions in front of the whole class. In addition, it offers the advanced learner prime-time assistance in his or her attempts to go beyond the basic requirements of the course.

Dividing the Elements— Multiplying the Time

Optimal use of class time is a crucial factor in success for both teacher and student. The fifty-minute period must be carefully divided so that no necessary elements of instruction are omitted. The essentials, with their approximate time frames, are listed sequentially below:

1. Pass out graded and recorded work from the previous session and answer any questions, reinforce areas of weakness, and assign remedial work if applicable (4 min.).
2. This will lead naturally into a review of recently learned concepts (3 min.).
3. The review will provide reinforcement. To ensure that the class stays on target with the master plan, spend some time on an overview. This will add meaning and credence to the present objectives (2 min.).
4. Because each student already has an assignment sheet, simply point out what is due (1 min.).
5. Perhaps the most crucial part of the presentation has now arrived. Clearly and concisely outline how the new work is to be organized, the manner in which the problems will be solved, and what form the solutions should take. As the introduction of the next lesson proceeds, carefully observe student reactions and questions, which will

(To page 39)

Under the section, "Nature and Purposes of the Tests," the authors of the Iowa Tests of Basic Skills make this significant comment: "Tests are aids to better instruction. As with other such aids, the usefulness of tests will depend upon the extent to which the test results are interpreted with wisdom, ingenuity, and caution. Test results should be used to supplement, not to replace, teacher judgment."

A careful study of standardized tests and a knowledge of each individual child's strengths and weaknesses in math will enable the teacher to plan sequential instruction that best addresses the child's needs and prepares him or her to participate in today's increasingly technological society. □

Scared of Math

(Continued from page 22)

the board when he solves the problem. Divide the class into three or four teams. The team that removes the most leaves for the week or the

month is the winner. This helps avoid lag time with students who complete the assignment before the end of the math period.

When the season changes, replace the leaves with nuts and change the caption to "Crack the Nuts," and use more difficult problems. Snowflakes may be used with the title "Catch It If You Can," and blossoms for spring with the caption "Spring Into Math."

The teacher should assemble a collection of problems at different levels of difficulty. The section entitled "Challenge for Able Students" that appeared between 1981 and 1983 in *The Arithmetic Teacher* has excellent problems that may be used for this purpose. Students can be invited to contribute problems from their daily lives or their reading.

Life is full of problems. Everyone will surely encounter them in every part of his or her life. As teachers we must prepare our students for life—prepare them to meet problem situations and not

be overcome by them. If we have helped students not to be afraid to take risks, not to be fearful of unfamiliar tasks, not to be ashamed of failure but consider it a steppingstone to success, not to glory in success—realizing mightier tasks are ahead of them, then we will have done our part. If we have not yet totally reached our goal, we can be comforted by the knowledge that if we have done our part, the Lord will always make up for our deficiencies and fill in the gaps where we have failed. □

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Lively Math Teaching

(Continued from page 19)

indicate when to change to one-on-one instruction (10 min.).

6. When it becomes apparent that most of the students understand how to proceed, devote the remaining time to the general practice session. During this time I devote myself entirely to individualized or small-group instruction. All grading and organizational work are put aside until later.

In conclusion, you must be prepared and inspired if you expect to be able to inspire others. If you, as a role model, transmit your internal excitement about your subject



to your students, they will in turn be inspired to set lofty goals and achieve great things. □

Individualized Instruction

(Continued from page 16)

nosis is necessary in order to deal efficiently with individual preferences and backgrounds. Since mathematics education has a spiral organization, children need to acquire basic skills before they are prepared to move on to higher-level skills. Failure at a single point can prevent further advancement. For remedial purposes, the teacher needs to precisely determine each student's abilities and weaknesses.

Some sources suggest that pretests and posttests should include similar questions, and if students display mastery of the subject they should be channeled into alternative activities. However, this is not usually recommended. Using long division again as an example, it is clear that a pretest using long division symbols and ideas would reveal little about the present status of most students. Rather, a pretest should be designed to reveal student mastery of requisite skills such as multiplying, subtracting, and recognizing the size relationship among numbers. Preassessment allows a teacher to intelligently prescribe a self-learning capsule, counsel a child regarding the scheduling of time, and decide whether to use a firm or gentle touch in dealing with a particular student problem.

Diagnostic Groups

Following formal preassessment the model shows individualization resulting in three diagnostic groups—one group needing enrichment, another needing practice, and a third needing special direction by the teacher. This grouping should

not be interpreted as representing levels of mastery, with the group needing the teacher being in some way inferior. Along the path entitled "teaching," the group may consist of boys one day and girls the next. At one time it may cater to the slow learner, and another time the fast learner.

Every student needs experiences afforded by each path, although not necessarily for each objective. In a small class the suggested structure provides for considerable breadth, for it will be almost impossible to prevent students in one group from knowing what happens in the others.

Enrichment

This path uses games and activities that will enrich the student's mathematical background and provide depth and breadth. The activities are designed to stimulate interest, arouse curiosity, and help maintain a high level of motivation. Of particular value are open-ended discovery-type experiments. Both independent and group projects can be utilized.

Children can be encouraged to conduct library research or work on favorite projects. They may prepare oral or written reports or search for jokes and cartoons that illustrate mathematical thought or problems. Under certain situations construction of geometric models will be appropriate. Some students will enjoy the creation of bulletin boards, exhibits, and other learning aids. What student could make a geoboard without learning something about measurement and asking how to use it? Often children from this group can assist students in the same or lower grades.

Unfortunately, this activity has often been restricted to the fast or bright student. In my classes, I have made a special point of training slower students to solve the

more difficult problems and then asking them to assist others. As the student explains the problems to other students, his or her learning is reinforced by the need to repeat the experience 20 or more times. This approach has helped improve the self-image of slow students, as they gain a sense of accomplishment from helping others.

Along the enrichment pathway students show creativity as they demonstrate ways they can best contribute to their own learning. Learning centers, learning packages, math labs, computers, and teaching machines are of particular value along this path. Although a beginning teacher may be frustrated in attempting to gather the quantity of material required for this pathway, he or she should not despair, since these materials can easily be exchanged between teachers or obtained from media centers.

Practice

The practice group may need activities designed to exercise the desired skill or provide practice in a particular prerequisite skill. Modules or computer-assisted instruction, workbooks, and practice sheets provide individualization. Modern commercial manipulatives, games, and workbooks are motivational and fun-oriented. Peer-group and teacher-made games, puzzles, stories, and books also have their places. Programmed learning packages are especially valuable.

To give immediate feedback and avoid the chore of record keeping, self-checking procedures can be developed. Rather than using a published answer book, why not have complete samples of exemplary student work? One alternative to tedious record keeping is to use student assistants. This teaches responsibility and attention to