Revitalizing the Mathematics Learning Environment

BY KENNETH L. SHAW

Mathematics teachers often fail to ask one crucial question: "What is mathematics?" When asked to define mathematics, they often say something like this: "Memorizing formulas and rules." "What is done in mathematics class," "What is in the textbooks," or "Getting the right answer when doing addition, subtraction, multiplication, and division." These descriptions convey a very narrow and static view of mathematics.

If we are to make significant, worthwhile changes in mathematics learning and teaching, we must first reflect on what mathematics is. This article uses a dynamic and process-oriented definition of mathematics to discuss the essentials of the mathematics learning environment.

A broader definition of mathematics is this: *Mathematics is the activity of constructing patterns and relationships.*[1]

Time for Change

Calls for more student-centered activity have come from a variety of sources.[2] These recommended changes for K-12 content and teaching call for radically different roles for students and teachers. The National Research Council advocates that teachers encourage active student engagement with mathematical ideas through both actions and thoughts, rather than having them passively listen to lectures.

Teachers themselves need experience in doing mathematics—in exploring, guessing, testing, estimating, arguing, and proving—in order to develop confidence that they can respond constructively to unexpected conjectures that emerge as students follow their own paths in approaching mathematical problems. Too often, mathematics teachers are afraid that someone will ask a question that they cannot answer. Insecurity breeds rigidity, the antithesis of mathematical power.[3]

To allow students the autonomy to develop this mathematical power, teachers should focus on student understanding, rather than overemphasizing the obtaining of correct answers.

This point is vividly illustrated through interviews with two fifth-grade students who were good at calculation, but lacking in understanding. Betty, the "top" student in the class, was asked to

multiply 293 and 54. She solved the problem quickly and flawlessly, but could not tell when the problem might be used.

Similarly, another "good" student solved the problem correctly using standard procedures, but gave the following application: "If I had a bag of potatoes that cost $2.93, and I bought bubble gum and it cost 54 cents, how much would I have altogether?"[4] Since he did not realize that he had given a wrong answer, the teacher simplified the question: "Can you give an aplication problem for 5 x 2 = 10?"

After much deliberation, he said, "If I had 2 baseball cards and someone gave me 5 more [long pause]; no, that would be 7." He tried several more attempts without success.

These students had good comprehension[5] of how to *do* the mathematics, but did not understand *why* they were learning it. The teacher defended her students by stating that this was the time to learn the fundamentals; later when they went to college, they could learn the applications.

The time for change is *now*. We must help all students understand the usefulness of mathematics in their world. They need to appreciate its interconnectedness and interesting aspects. To accomplish this, we must create a new learning environment.

Learning Environment

To revitalize mathematics learning and emphasize student learning and mathematical power, teachers must concentrate on the meaning of knowledge, teaching, and learning.

Knowledge

Knowledge is formed by making sense of one's experiences. Just because a teacher tells students how to solve the problem does not mean that they necessarily have the knowledge to do so. The pipeline metaphor that depicts the teacher pouring knowledge into the students does not work. Students must actively assimilate into their past knowledge whatever new information the teacher is sharing. This knowledge is based on their experiences.[6] In reality, mathematical knowledge is individualistic and personally constructed, since students have a wide variety of backgrounds and experiences.[7] When we accept this view of knowledge, teaching and learning will look much different.

Picture Removed

If we are to make significant, worthwhile changes in mathematics learning and teaching, we must first reflect on what mathematics is.

Teaching and Learning

Teaching and learning are complex phenomena that cannot be reduced to a set of rules. "Teaching mathematics draws on knowledge from several domains: knowledge of mathematics, of diverse learners, of how students learn mathematics, of the contexts of classroom, school, and society."[8] Consequently, teachers should concentrate on two main areas: providing significant and worthwhile tasks, and engaging students in meaningful dialogue.

Mathematical tasks should be designed to pique student interest and engage them in negotiation, exploration, conjecture, testing, and justification. These activities should often occur in small-group settings where students feel free to share their ideas with their peers.[9]

Etchberger and Shaw[10] share the story of Jessica, a fifth-grade teacher who changed from an instrumental teacher-transmitter whose students were mere receivers, to a relational teacher-provider whose students were cooperative constructors. Jessica had taught fraction operations in a very procedural way until she realized that her students were not making sense of the concept; they were just memorizing their way through the unit.

Jessica decided to set aside the textbook and do an activity on equivalent fractions. After having her students form groups of four, she gave each group one sheet of paper. She asked them to fold the paper and shade half of it. Next, the students were to fold the paper again and write a fraction that was equivalent to 1/2 (namely 2/4).

Jessica asked the groups to repeat

this process and to identify as many fractions as they could that were equivalent to 1/2. Most groups quickly found a pattern and quit folding their papers. Jessica was astonished at how quickly they discovered the patterns and how well they could describe their processes to the whole class. She then assigned another task to determine whether students could explain why the following fractions were not equivalent to 1/2: 11/23 and 1362/3000.

Jessica remembers: "They all shouted at one time. They were into this! There was not one person sitting back." They all wanted to share why the fractions were not equivalent to 1/2.

This task was exploratory for the students; they had to find patterns, describe their processes and defend their answers. And Jessica no longer saw herself as answer giver. Her role now was to provide an environment in which students felt free to discuss, argue, elaborate, and clarify their thoughts. Jessica recalls the last activity of the lesson:

Picture Removed

"I held up the sheet of paper, folded and shaded and asked them in my most disbelieving voice that if the paper was folded in 5,876 tiny rectangles would there be 2,938 rectangles shaded? I told them I didn't believe it. They insisted that it was so and argued with me using all the logic they could muster. This was good stuff."

Jessica's teaching has been energized through her use of a more student-oriented approach. She views teaching much differently now. She uses her textbook as a resource, not a guide; she spends time developing tasks that will help students construct knowledge that will be meaningful to them; she listens and learns from her students what knowledge they have constructed. With this information she develops new tasks that will produce even greater understanding of the mathematics concepts.

Jessica now recognizes the difference between disruptive noise and constructive noise; she realizes the value and necessity of having students learn from one another. Most important, she believes in what she is doing and feels empowered to make good decisions to improve the learning environment of her students.

Conclusion

Typically, mathematics classes have focused on getting the right answer. Often, the right answer must be obtained by using the teacher's method; all other ways are considered wrong. No wonder many students feel great anxiety about mathematics.

The mathematics education community has recently spearheaded one of the greatest mathematics reform efforts in our country's history. The National Council of Teachers of Mathematics Curriculum and Evaluation Standards of School Mathematics[11] has served as the impetus for change in mathematics at all levels. Following these standards, many of the new textbooks emphasize the use of technology, manipulatives, applications, cooperative learning, and problem solving.

Teachers can help create a new learning environment in their classrooms by reflecting on their own practices, seeking ways to improve their teaching and learning, and making mathematics constructive and lively. This will create a revitalized classroom mathematics learning environment for their students.

Dr. Kenneth L. Shaw is an Assistant Professor of Mathematics Education at Florida State University, Panama City Campus, in Panama City, Florida. He has taught mathematics, physics, and computer programming at the academy level, and holds a doctorate in mathematics education with specialty in teacher education. He co-directs the Enhancement of Mathematics and Science Teaching Project at Florida State University, provides in-service for elementary and secondary teachers, and co-directs evaluation for the implementation of the Comprehensive Plan to Improve Mathematics, Science, and Computer Education for the state of Florida.

Calls for more student-centered activity have come from a variety of sources.

REFERENCES

1. Grayson Wheatley, *MAPS Plus: Mathematics Achievement Through Problem Solving* (Tallahassee: Florida State University, Mathematics Education Department, 1992).

2. Mathematical Sciences Education Board and National Research Council, *Reshaping School Mathematics: A Philosophy and Framework for Curriculum* (Washington, D.C.: National Academy Press, 1990); National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics* (Reston, Va.: NCTM, 1989); *Professional Standards for Teaching Mathematics* (Reston, Va.: NCTM, 1991).

3. National Research Council, *Everybody Counts* (Washington, D.C.: National Academy Press, 1989), p. 65.

4. Kenneth L. Shaw, *Contrasts of Teacher Ideal and Actual Beliefs About Mathematics Understanding: Three Case Studies.* (Unpublished doctoral dissertation, University of Georgia, Athens, 1989).

5. Richard R. Skemp, "Relational Understanding and Instrumental Understanding," *Mathematics Teaching* 77 (1976), pp. 20-26; *The Psychology of Learning Mathematics* (Hillsdale, N.J.: Lawrence Erlbaum Press, 1987).

6. Ernst von Glasersfeld, *Constructivism as a Scientific Method* (Oxford: Pergamon Press, 1987).

7. Tom Kieren, "Children's Mathematics/ Mathematics for Children: Notes on Mathematical Environments for Young Children." In *Transforming Early Childhood Mathematics Education: An International Perspective*, L. Steffe and T. Wood, eds. (Hillsdale, Ill.: Lawrence Erlbaum Press, 1990); Leslie Steffe, "Overview of the Action Group A1: Early Childhood Years." In Steffe and Wood, ibid.; Grayson Wheatley, "Constructivist Perspectives on Science and Mathematics Learning," *Science Education* 75:2 (1991), pp. 9-21.

8. National Council of Teachers of Mathematics, 1991, p. 22.

9. Elizabeth Cohen, *Designing Groupwork: Strategies for the Heterogeneous Classroom* (New York: Teachers College Press, 1986); Neil Davidson, *Cooperative Learning in Mathematics* (Menlo Park, Calif.: Addison-Wesley, 1990); David Johnson, Roger Johnson, and Edythe Johnson Holubec, *Circles of Learning: Cooperation in the Classroom* (Edina, Minn.: Interaction Book Co., 1986); Erna Yackel, Paul Cobb, Terry Wood, Grayson Wheatley, and G. Merkel, "The Importance of Social Interactions in Children's Construction of Mathematical Knowledge." In *1990 Yearbook of the National Council of Teachers of Mathematics*, Tom Cooney, ed. (Reston, Va.: National Council of Teachers of Mathematics), pp. 12-21.

10. Mia Lena Etchberger and Kenneth L. Shaw, "Teacher Change as a Progression of Transitional Images: A Chronology of a Developing Constructivist Teachers," *School Science and Mathematics* 92:8 (December 1992), pp. 411-417.

11. National Council of Teachers of Mathematics, 1989.