Teaching Number Sense in the Elementary School

BY ROBERT C. MOORE

Picture Removed

Christy Shares Her Cookies, and Other Stories

At the supper table one evening my daughter, Christy, then six years old, told about going to the grocery store and getting free cookies for herself, her sister and brother. Asked if Mom got a free cookie too, she said No. Not expecting a reply, I wondered aloud how four people could share three cookies equally. After a long pause, to my surprise Christy replied, "The way I would do it is I would cut these two cookies like this [indicating cutting each in half] and then I would cut the other cookie like this [indicating cutting it into quarters]. Then I would give everyone a big piece [a half] and a small piece [a quarter]." Christy's solution showed the depth of her understanding of geometry and fractions, as well as the creativity children can apply to mathematics problems when given a chance.

Tom Carter was making a circular flower bed in his yard. In trying to determine how much fencing he needed to put around it, he tried to multiply mentally, using 3.14 for π until his father pointed out, "Tom, for practical purposes pi is 3!" It had never before occurred to him that three might be an adequate approximation of π; rather, he dutifully used 3.14 as he had learned in school.

A news magazine reported that in trying to compete with the 1/4-pound hamburger of another restaurant, one fast-food chain offered a 1/3-pound hamburger for a few cents more. But the larger burger did not sell well, apparently because customers thought 1/3 was *smaller* than 1/4.

What Is Number Sense?

Number sense is a recurring theme in the many recent calls for educational reform. Although it is difficult to define number sense, the stories recounted above will sound familiar to teachers who have witnessed number sense—or its lack—in their classrooms and communities.

In her book *Developing Number Sense*, Reys writes:

Number sense refers to an intuitive feeling for numbers and their various uses and interpretations; an appreciation for various levels of accuracy when figuring; the ability to detect arithmetical errors; and a common-sense approach to using numbers. . . . Above all, number sense is characterized by a desire to make sense of numerical situations.[1]

A person with good number sense will look at a mathematics problem holistically and choose a strategy that matches the context of the problem and the level of accuracy desired, rather than simply jumping into the problem and using a standard procedure. For example, at a restaurant such a person might mentally compute a 15 percent tip on a $28.63 bill by computing 10 percent of the bill, which is a little less than $3.00, and an additional 5 percent, which is about half of $3.00, then adding them together ($3.00 + $1.50) to get $4.50. In the episodes above, Mr. Carter's father showed good number sense by using 3 as an estimate for π, and Christy found a common-sense way to share the cookies by thinking geometrically.

These kinds of intuitive and creative use of numbers would not occur to many elementary school students, who have learned standard addition, subtraction, multiplication, and division procedures to be done with paper and pencil. They become hampered by their desire to "follow the rules."

Calculation skills are important. But students also need a broader understanding of numbers. The *Curriculum and Evaluation Standards for School Mathematics* provide a more precise definition of number sense:

Children with good number sense (1) have well-understood number meanings, (2) have developed multiple relationships among numbers, (3) recognize the relative magnitudes of numbers, (4) know the relative effect of operating on numbers, and (5) develop referents for measures of common objects and situations in their environments.[2]

Number sense is a recurring theme in the many recent calls for educational reform.

I have used these five aspects of good number sense to organize the activities listed below. You may want to refer back to this list as you read through the activities.

Teaching for Number Sense

Teaching for number sense is not the same as teaching paper-and-pencil skills. It requires a learning environment that encourages students to make sense of numbers by manipulating concrete objects, investigating numerical situations, estimating answers, and discussing their findings. It focuses not so much on correct answers as on the students' solution strategies and ways of thinking about the problem. It is not something for the teacher to try to squeeze into the curriculum. Rather, it is an approach that should permeate mathematics teaching.

The following activities illustrate

some ways to help students develop good number sense. They can be adapted for various grade levels and should be repeated over a period of time for maximum benefit.

Activity 1: Incredible equations.[3] This activity will help students understand number meanings and relationships.

Choose a number—say 24. Ask the students what it makes them think of. (Possible answers include 24 hours in a day, 24 months in two years, two dozen eggs, or one penny less than a quarter.) Now have them write some "incredible equations" for 24 on the chalkboard or bulletin board. Encourage them to be creative. Here are some examples:

$24 = 2 \times 12$
$24 = 25 - 1$
$24 = (100 \div 4) - 1.$
$24 = 2 \times 2 \times 2 \times 3$
$24 = 2 \times \sqrt{100} + 4.$

You can vary the activity by choosing the day of the month for the number, or by challenging the students to find equations that use all four operations (+, –, x, ÷). With practice they will become more flexible and creative.

Activity 2: Broken Key Problems.[4] These problems will give students a better understanding of place value and, like Incredible Equations, greater insight into the many relationships among numbers. The problems also help students understand the effects of addition, subtraction, multiplication, and division.

Give each student a calculator. Tell them to pretend the 3 key is broken. How can we do these addition and subtraction problems without pressing the 3 key?

327	87	281	836
+183	–53	–38	–388

When Ms. Lauritzen asked her fourth graders to try to enter 327 into their calculators without pressing the 3 key, John began by entering 1 + 1 + 1, rather than 100 + 100 + 100, whereas Karen entered 299 + 28. The exercises revealed great differences in these students' understanding of place value and numerical relationships, differences that were less apparent in their regular mathematics homework.

Picture Removed

A person with good number sense will look at a mathematics problem holistically and choose a strategy that matches the context of the problem and the level of accuracy desired.

Ask the students to play another game. Their score is the number of times they press the addition, subtraction, and equals keys. The person with the lowest score wins. When Mr. Hall challenged his sixth graders with these problems, Benjamin first changed 327 + 183 to 227 + 283 by taking one from the 3 in 327 and putting it with the 1 in 183, and then changed the problem again by taking one from the 3 in 283 and putting it with the 7 in 227, thus arriving at 228 + 282, which has no 3s. Thus he was able to get the answer by pressing the addition and equals keys only once. To subtract 38 from 281, Shelly used a similar strategy. She reduced both the 8 and the 3 by one to convert the problem to 271 – 28. The students found that in order to leave the answer unchanged in an addition problem, they had to increase a digit in one number while decreasing one in the other number, whereas in a subtraction problem they had to increase or decrease digits in both numbers.

Extending these games to multiplication and division can lead to some inter-

esting strategies and discussions.

Activity 3: Clue Game. This activity will help develop students' understanding of number meanings. It also reviews some terms for classifying and comparing numbers.

Think of a number. Ask the students to try to discover the number from the clues you give. Give only one clue at a time, and ask the students to write a list of possible answers after each clue.

> **Clue 1. The number is less than half of 80.**
> **Clue 2. The number is more than 10.**
> **Clue 3. The number is odd.**
> **Clue 4. The tens digit is 3.**
> **Clue 5. The ones digit is less than 5.**
> **Clue 6. The tens digit plus the ones digit equals 6.**
> **Answer: 33.**
>
> **I have some money in my pocket. There are some dollars, some dimes, and some pennies.**
> **Clue 1. The amount of money is more than half of $10.00.**
> **Clue 2. If you double $4.00, you will have more than I have.**
> **Clue 3. The number of whole dollars is the same as 20 quarters.**
> **Clue 4. There are 7 dimes.**
> **Clue 5. The pennies are worth 2 cents less than a nickel.**
> **Answer: $5.73.**

Ms. Lauritzen's fourth graders enjoyed choosing a number and making up their own clues about it.

No Answer Please[5] is a similar activity. Students are to develop clues or facts about the answer to a problem without computing or telling what the answer is. Be sure to have the students give reasons for their responses. For example, 276 + 121 =_____. Here are some clues about the answer that students might give:

The answer is more than 300 because 200 + 100 = 300.

The answer is less than 400 because 76 + 21 is less than 100.

The answer is an odd number, because 6 + 1 is odd.

Activity 4: Number Strips and Slides. These activities will help students recognize the relative sizes of numbers from 0 to 100, 0 to 1000, 0 to 1, or whatever range you choose.

Give each student two or three strips of paper about 10 inches long and 1 inch wide. Ask the students to write 0 at one end of the strip and 100 at the other end. Then ask them to mark 50. Some students will probably fold to find the middle, so have them share this method with the others. Next, ask them to label 25 and 75. How can we fold to find where these numbers go? Now ask them to label other numbers on the strip. For example:

45	How close should it be to 50?
80	It should be close to 75.
10	We pay 10 cents tithe on each dollar.
15	A restaurant tip is 15 cents per dollar.

Watch where the students place the numbers on their strips. This will tell you how they perceive the relative sizes of numbers. You can then move on or reteach as necessary.

Make a paper "Number Slide." Cut two strips of paper about 10 inches (25 cm) wide–one white strip and one of another color. Glue the strips together to make one long strip. Cut two horizontal slits about 8 inches apart in a sheet of white paper. Label one slit 0 and the other 100. Insert the long strip through the two slits. As you pull the strip up and down, the colored part rises and falls like the mercury in a thermometer.

Move the colored strip to various positions, and ask the students what number they think it marks. Ask them to explain their reasoning.

Now reverse the question. Give the students numbers and ask them to pull the strip to show the number. For example, tell the class to think of the number slide as a Celsius thermometer. Ask the students to show 37°C (body temperature) and 22°C (room temperature).

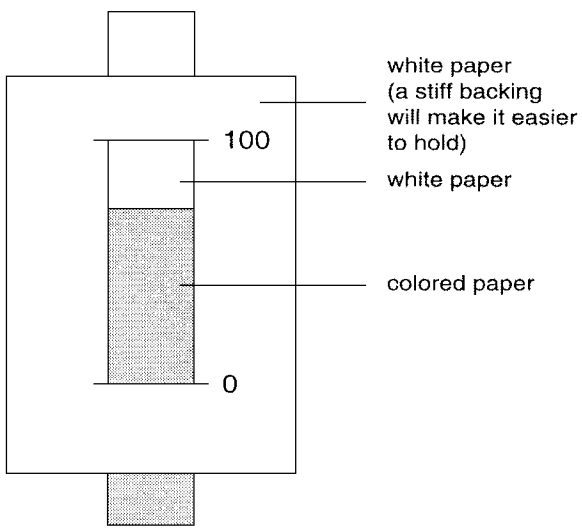
By labeling the slits 0 and 1, or 0 and 2, you can use the slide to help your students develop meanings for fractions, decimals, and percents as well.

Activity 5: Comparing Numerical Expressions.[6] This activity helps students think about numbers, operations, and the way changing one number affects the other numbers in a numerical statement.

On the chalkboard or overhead projector, write a pair of numerical expressions. Put a box between them. Give each student three different-colored cards, say green, yellow, and pink. Have them write the symbol > and the words "greater than" on the green card, the symbol < and the words "less than"

Figure 1

Number Slide



on the yellow card, and the symbol = and the word *equals* on the pink card. Tell the class that one of the signs on their cards belongs in each box. When you say "Go!" they are to hold up a green card if the first expression is *greater than* the second, a pink card if the first expression is *less than* the second, or a yellow card if the expressions are *equal*. Ask them to share their reasoning. Here are some expressions to try:

1. 186 + 493 + 278 ❑ 192 + 501 + 283
2. 72 + 73 + 74 ❑ 3 x 73
3. 250 ÷ 6 ❑ 250 ÷ 8
4. 30 x 13 ❑ 31 x 12

When I presented item 2, Jared said they are equal because "if you take one from the 74 and put it with the 72, then you have 73 + 73 + 73. So they're equal." Item 3 can provoke discussion about the effects of division and how the sizes of the numerator and denominator affect the quotient. Item 4 can lead to some interesting investigations about multiplication and the effects of factors.

Activity 6: Maximum Product Game.[7] Like the preceding activity, this one will stimulate discussion about multiplication, particularly its relationship to place value. Be prepared for some surprises!

Ask the students to place the digits 1, 2, 3, 4, and 5 on the five blank lines in Figure 2 in order to produce the largest possible product. They are to use each digit only once.

Figure 2

_____ _____ _____

x _____ _____

_____

After they fill in the diagram, have them discuss why they placed the digits as they did. Then have them check their results with a calculator. You might also ask them to arrange the digits to produce the smallest product, or to produce as many different products as possible.

Now play a game. The object is to get a larger product than the other players. To play the game, roll a die (or spin a spinner) and ask the students to insert the number that comes up on one of the five blank lines in the diagram. Keep rolling the die until the diagram is filled, one digit at a time, then have them compute their products. The students with the highest product win. You may wish to arrange the students in pairs and have them play against each other. Be sure to have students describe the strategies they use. Notice that judgments about probability and chance arise in the game.

Teaching for number sense is not the same as teaching paper-and-pencil skills.

Activity 7: Golf Game.[8] This challenging game will help your students to better understand decimals and the effects of multiplying by numbers less than or greater than 1. It is also an excellent way to practice estimation with multiplication.

Start with a "ball," say 18, and try to hit it into the "hole," say 840. Let's call this a par 4 hole, so we get to hit the ball 4 times. We choose a "club," say 42, to multiply by 18: 18 x 42 = 756. Without clearing the calculator display, we must now hit the ball from where it is; that is, we must choose a club to multiply by 756 to get 840. Let's try 1.2. Since 756 is already displayed on

the calculator, we simply press "x 1.2" and then the equals key to get 907.2. Again, without clearing the display, try .9: 907.2 x .9 = 816.48. For our fourth hit, try 1.05: 816.48 x 1.05 = 857.304. The winner is the person whose ball is closest to the hole.

To help the students improve their estimation skills, discuss strategies. Begin by asking them to share the rounding and estimation strategies they already use. You can also talk about using 10 percent and 1 percent. For example, in the game above, when we had 756, multiplying by 1.1 (rather than 1.2) would have added one-tenth, or 10 percent, of 756 onto 756. Since 10 percent of 756 is about 75 and 756 + 75 is about 750 + 75, or 825, 1.1 would have been a good club: 756 x 1.1 = 831.6. Now multiplying by 1.01 will add 1 percent to 831.6, which is about 8: 831.6 x 1.01 = 839.916.

To vary the game, have the students play in pairs against each other or against other teams, or let them challenge you. Instead of restricting the number of hits, set a time limit or have them keep hitting the ball until they get within a certain interval about the hole, like 830 to 850.

Conclusion

It has been my experience that teaching number sense is well worth the effort. My own number sense has improved along with my students', and we have all had fun. But more importantly, the students' attitudes about mathematics have changed. Some now see that doing mathematics does not mean simply following the rules; it also means thinking creatively, inventing one's own strategies, and making sense of numbers and patterns in unique ways. If students develop positive attitudes about mathematics and their ability to think numerically, they will be empowered to succeed in mathematics and apply it successfully and confidently in their lives.

The author wishes to thank Nicole Corrigan and Elisabeth Philbrick for their help in developing these number sense activities, and Tom Hall and Vicky Lauritzen of A. W. Spalding Elementary School, Collegedale, Tennessee, for trying the activities in their classrooms.

Picture Removed

Robert C. Moore is Professor of Mathematics at Southern College of Seventh-day Adventists, Collegedale, Tennessee. Currently he is working with the mathematics education program as a visiting professor in the Department of Mathematical Science at Montana State University, Bozeman, Montana. Dr. Moore has worked as a mathematics and science teacher at the academy and collegiate level, has taught elementary and secondary mathematics methods classes, and has worked with elementary teachers in Georgia and Tennessee in the areas of geometry and measurement and number sense.

Additional Resources

Bitter, G. G. and J. L. Mikesell. *Using the Math Explorer Calculator: A Sourcebook for Teachers.* Menlo Park, Calif.: Addison-Wesley, 1990.

Calculators [Focus issue]. *Arithmetic Teacher* 34:6 (February 1987).

Coburn, Terrance G. *How to Teach Mathematics Using a Calculator: Activities for Elementary and Middle School.* Reston, Va.: National Council of Teachers of Mathematics, 1987.

Fey, James T. and Christian R. Hirsch. *Calculators in Mathematics Education: 1992 Yearbook.* Reston, Va.: National Council of Teachers of Mathematics, 1992.

Number Sense [Focus issue]. *Arithmetic Teacher* 36:6 (1989).

Reys, Barbara J. *Developing Number Sense in the Middle Grades: Addenda Series, Grades 5-8.* Reston, Va.: National Council of Teachers of Mathematics, 1991.

Schoen, Harold L. and Marilyn J. Zweng. *Estimation and Mental Computation: 1986 Yearbook.* National Council of Teachers of Mathematics, 1986.

Trafton, Paul R. and Albert P. Shulte. *New Directions for Elementary School Mathematics: 1989 Yearbook.* Reston, Va.: National Council of Teachers of Mathematics, 1989.

REFERENCES

1. B. J. Reys, *Developing Number Sense in the Middle Grades: Addenda Series, Grades 5-8* (Reston, Va.: National Council of Teachers of Mathematics, 1991), pp. 3, 4.

2. National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics* (Reston, Va.: National Council of Teachers of Mathematics, 1989), p. 38.

3. Hilde Howden, "Teaching Number Sense," *Arithmetic Teacher* (February 1989), p. 8.

4. Ibid., pp. 8-10.

5. David J. Glatzer and Joyce Glatzer, "No Answer Please," *Arithmetic Teacher* (February 1989), p. 38, 39.

6. Reys, 1991, p. 26.

7. Ibid., p. 25.

8. Barbara J. Reys, "The Calculator as a Tool for Instruction and Learning," in Paul R. Trafton and Albert P. Shulte, eds., *New Directions for Elementary School Mathematics: 1989 Yearbook* (Reston, Va.: National Council of Teachers of Mathematics, 1989), pp. 171, 172. Reys calls this game TARGET.