Contemporary Ideas for Teaching Secondary-Level Mathematics

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Introduction

Because of the efforts of the National Council of Teachers of Mathematics and other groups, secondary mathematics education is entering a period of dramatic reform. For example, students are using technology and working cooperatively to solve real-world problems, and teachers are focusing on "new" content (such as mathematical modeling and statistics) and guiding students' mathematical exploration. As never before, classroom emphasis is on the processes that lead to mathematical under-standing. In this article, we offer some contemporary ideas about the teaching of mathematics in the secondary school, as well as some thoughts about the role of faith in this process.

Encourage Students to Become Active Learners of Mathematics

In mathematics teaching, as in other areas, we can learn much from the techniques of the Master Teacher. In His efforts to teach us, the Lord uses a variety of methods. In some cases, He communicates to us directly, providing us with the knowledge that we need to make intelligent decisions. However, the Lord also allows us to experiment, to in-vestigate, and to learn from our mistakes. He understands that in some situations, experi-ence is the best teacher.

As secondary teachers, we can follow the Lord's example by using a variety of instruc-tional methods. In addition to sharing our expertise directly, we can provide our stu-dents with tools and activities that allow them to explore and "discover" mathematics for themselves. For instance, students can learn mathematics through the use of manipula-tives, calculators, computers, games, coopera-tive learning, real-world modeling, and prob-lem-solving investigations.

An Exploratory Approach

The following activities exemplify these al-ternative methods of teaching. As you imple-ment these ideas, be sure to allow ample time for students to complete each activity, and be careful not to confine their attention to the particular concepts you have in mind. Remember that the underlying objective is to encourage students to conjecture, experiment, and discover mathematics for themselves.

• Have students use the *Geometry Supposer* or a related software package to construct a triangle and connect the midpoints of any two sides. Ask them to determine the slope and length of the resulting line segment, and to compare these with the length and slope of the third side of the triangle. Have them dis-cuss their findings. Do they think their re-sults hold true for any triangle? Have them construct an appropriate theorem.

• Introduce the concepts of exponential functions and half-life by having students toss 100 coins, remove those that come up tails, and record the number that remain. Repeat this process until most of the coins have been removed. Then have them graph the data and construct a model that allows them to predict the number of coins that will remain after any given number of tosses.

• Have students use graphing calculators to examine the changes that occur in the graph of $Y = AX^2 + BX + C$ as the values of A, B, and C vary.

Place Students in the Role of Teacher

Psychologists claim that we remember 10 percent of what we hear, 20 percent of what we see, but 90 percent of what we teach. Thus, an effective teaching strategy is to place students in the role of teacher. Just as we grow spiritually by sharing our faith, so our students deepen their mathematical understanding by sharing their insights with others.

One way to place students in a teaching role is through cooperative learning. In general, cooperative learning means assigning tasks to groups, rather than to individuals. Ideally, each group should find a solution to the assigned task and communicate the findings to the class. To ensure that all group members actively participate, inform the groups that you will randomly select a person from each group to present the group's solution to the class. As they prepare all group members to present a solution, students function as teachers. As one member of the group discovers a step in the solution process, he or she must communicate the idea to others in the group. Furthermore, as they share their solution with the class, groups actually teach other groups.

A key to the success of cooperative learning is careful task selection.

Cooperative learning tasks should encourage collaboration among group members. The following tasks are ones that we have used successfully in our classes:

• Distribute a loop of string about two meters long to each group. Ask each group to use only protractors in shaping the string into the following polygons: equilateral triangle, square, regular pentagon, or regular hexagon. In the process, students must devise strategies to ensure that both the angles and the lengths of the sides are equal. Have each group share their solution strategies with the class.

• Assign each group a topic from a book or journal, such as Euclid's proof of the Pythagorean Theorem or Heron's formula for the area of a triangle. Have them prepare a short presentation and a one-page summary of the topic for the other members of the class.

Emphasize the Connectedness of Mathematics

We gain a complete picture of God's love only as we study His Word as a whole, comparing scripture with scripture. Compartmentalization obscures the harmony.

In the United States, the teaching of mathematics has traditionally been compartmentalized. We have separate classes for algebra, geometry, and so on. However, the topics covered in these courses are not unrelated. For example, geometry topics can reinforce our understanding of algebra, and vice versa. Moreover, by understanding the connections between mathematical topics, students can quickly construct (and translate between) alternative representations of problem situations. This understanding empowers students as problem solvers.

In teaching mathematics, we need to include activities that build connections between the areas of mathematics. For example, the following activities encourage students to explore different ways that mathematics can be used to describe a situation or accomplish a task:

• Have students explain how the following ideas are related: the Pythagorean Theorem, the standard equation of a circle ($x^2 + y^2 = r^2$), and the trigonometric identity $\sin^2 x + \cos^2 x = 1$.

• Have students use synthetic *and* algebraic techniques to prove that the diagonals of a rectangle bisect one another.

• Have students reflect, translate, and rotate geometric figures using dot paper, tracing paper, Miras, and matrices.

Address the Cognitive and Affective Components of Learning

Our Lord is patient. He does not force us to accept Him, but waits for us to do so willingly. It is much the same with learning. Psychologists claim that in order to learn a piece of information, students must actively reorganize their existing knowledge structure to accommodate the new (and possibly conflicting) information. However, in order to do so, students must want to put forth the effort needed to reorganize their existing view of the world. If a student is unwilling to do this, he or she will not

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learn—regardless of what we do as teachers. In short, learning is both a cognitive and affective activity.

In the mathematics classroom the affective side of learning is particularly visible. Some students harbor negative or apathetic feelings toward mathematics. To them, mathematics is simply a collection of meaningless rules lacking any relevance to the "real world." As a result, these students have no desire to learn mathematics.

On the other hand, many students want to learn mathematics but simply lack confidence in their ability. Whether they can or cannot do mathematics is often beside the point. These

students believe that mathematics is inaccessible and, as a result, they make only half-hearted attempts to learn the subject.

Mathematics teachers can address these problems in several ways:

First, to build students' self-confidence, we must be patient with them, as our Lord is patient with us. Remember that learning is not a race. The objective is to help every student learn.

Second, to instill in students a desire to learn mathematics, we can involve them in solving real-life problems and, in the process, demonstrate how a comprehension of mathematics leads to a better understanding of our world.

Third, to empower students as learners of mathematics, we must assign tasks that are accessible to all students and allow each student to contribute to the accomplishment of the objective.

The following activities are designed to address students' affective (and cognitive) development:

• Have students list their hobbies or areas of interest. Have them interview experts in these areas to determine what mathematics is used.

• Assign students to groups. Have each group gather world population data and construct a model to predict the population in some specified year. Allow each member of the group to describe to the class some aspect of the solution process, such as the data collection, the graphing of the data, the construction or use of the model, or an evaluation of the reasonableness of the prediction.

Use a Variety of Assessment Methods

As Christians, our individual experiences and perspectives influence how we relate to the Lord and interpret His Word. He deals with us according to our individuality. Similarly, our students interpret what we say about mathematics according to their individual perspectives and experiences. The better we understand our students' mathematical thinking, the better equipped we will be to effectively teach them.

In addition to tests, quizzes, and homework, we can use a variety of assessment tools, such as those listed below, to probe our students' minds and redirect our teaching to fit their level of understanding. Furthermore, regular and frequent assessment will provide our students with feedback that enhances their learning.

• As an alternative to verbal give-and-take, have the students respond to your questions in writing. By quickly walking around the room and glancing at their papers, you can get valuable feedback from many students, including those who would not dare to call out an answer. Also, when students' answers differ, you may want to use them as springboards for discussion.

• About five minutes before the end of class, distribute an index card to each student. Ask them to write a brief explanation of the central topic of your lesson, such as the slope of a line. Have them list their questions about the topic. A quick reading of the cards will reveal much about the students' understanding and can guide your preparation for the next day's lesson.

• Have each student keep a journal about his or her progress, problem-solving attempts, frustrations, and questions, or a portfolio containing samples of the best work done during the course.

Become Active in the Mathematics Teaching Community

Fellowship among church members strengthens the church body. Through positive interaction, we are able to share our understanding of the Word and support one another's Christian growth. Similarly, fellowship among mathematics teachers enables us to grow in our profession. As we work cooperatively and share our ideas, successes, and failures, we support one another in professional growth. By participating in activities such as the following, we can benefit from the wealth of teaching talent in the field and contribute to the growth of our fellow teachers.

• Discuss effective teaching strategies with the teachers in your school or conference.

• Join an organization of mathematics educators, such as the National Council of Teachers of Mathematics or the Mathematical Association of America.

• Attend regional or national conferences of mathematics educators to share strategies or activities that have been successful in your classroom.

• Subscribe to at least one mathematics education journal, such as the *Mathematics Teacher* or *School Science and Mathematics*. Write an article for one of these journals.

Conclusion

In this article, we have offered a few ideas regarding the teaching of secondary mathematics, as well as some concrete implementations of these ideas. Naturally, this is not an exhaustive list of everything that is important in mathematics education at this level. It is simply meant to evoke interest and provide a glimpse of the current reform movement in our field. For additional information on these and other issues related to the teaching of mathematics, you may wish to refer to the following sources:

Cooney, T. J. and C. R. Hirsch. *Teaching and Learning Mathematics in the 1990s: 1990 Yearbook*. Reston, Va.: National Council of Teachers of Mathematics, 1990.

National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics*. Reston, Va.: National Council of Teachers of Mathematics, 1989.

National Council of Teachers of Mathematics. *Professional Standards for Teaching Mathematics*. Reston, Va.: National Council of Teachers of Mathematics, 1991. @

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