Hands-on Mathematics

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Front-page headlines in a recent edition of the *Los Angeles Times* blare "Debate Over How to Teach Math Takes Cultural Turn."[1] The 68-column-inch article echoes the heated debate between supporters of traditional methods of teaching mathematics and those supporting reform. Ever since *A Nation at Risk*[2] shocked America in 1983, the plight of mathematics education has made headlines. This report and its aftermath have prompted serious endeavors to remedy this situation. First, let's look at the scope of the problem worldwide and the two principal approaches taken to solve it. Then I will outline some of my efforts to improve the understanding and achievement of students in my college math classes.

At the forefront of the reform movement, the National Council of Teachers of Mathematics[3] (NCTM) urges:

- The use of methods that unify the various branches of mathematics;
- Greater emphasis on problem-solving;
- Group learning;
- Hands-on encounters with mathematical concepts;
- Reasoning and communication skills, as well as computational skills.[4]

The council has spent years creating comprehensive guides for teaching math from kindergarten through the 12th grade. The latest version, *Principles and Standards for School Mathematics*, published in the spring of 2000, is available electronically at http://standardse.nctm.org.

By contrast, traditionalists believe that expertise in mathematics comes from rote memorization of the basic facts followed by extensive work with formulas and equations, pointing out that the more successful programs in Asia and Europe use these techniques. They accuse the reformers of being bigoted, sexist, and racist for thinking that a change in teaching methods will be instrumental in improving the performance of underachievers.

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The author's elementary teachers-in-training use hands-on math to discover the Fibonacci sequence.

Unfortunately, neither side can produce hard facts proving that its stand is superior to its opponent's. As in most heated debates, the optimum course probably lies somewhere between the two extremes.

An International Problem

Poor mathematics education is not restricted to the United States. George Malaty of the University of Joensuu, Finland, sees a divide in mathematics education that he calls an Eastern School and a Western School. He uses the U.S. to characterize the West and Russia to represent the East. He says that "where changes have occurred in the West, they are likely to be of benefit to mathematical education, whereas the ongoing changes in the East are likely to prove to be rather more problematic both for the teaching of mathematics and for mathematics itself."[5]

Luis Moreno-Armella of Departmento de Matemática Educativa in Mexico City observes that "two related statements are increasingly accepted in educational circles: (1) the teacher's conception of mathematics determines to a great extent, the way (s)he teaches; and (2) mathematics should be conceived of as *an activity* and not (only) as a *product*."[6]

Matías Camacho of the University of La Laguna, Spain, points out that the Spanish Ministry of Education requires that a mathematics teacher be able (among other things) to "assume that 'mathematical activity' plays an essential role in the construction of mathematical thinking."[7]

The oft-quoted Chinese philosopher, Confucius, is credited with saying: "I hear and I forget, I see and I remember, I do and I understand."[8] Therefore, *doing* mathematics should involve much more than just paper and pencil exercises; it should incorporate the use of concrete objects to reinforce learning.

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Mathematics for Prospective Elementary Teachers

In the spring of 1993, I was assigned to teach a class for prospective elementary teachers, Concepts of Mathematics, at La Sierra University (Riverside, California). For the first two years, I used a very traditional approach. In the summer of 1994, however, I participated in an NSF-funded PROMPT[9] workshop at Humboldt State University. The participating professors had spent a lot of time rethinking how math is taught and creating activities that would enhance learning. These ideas started me in a new direction.

Over the past six years, I have tried to incorporate both reform and traditional methods into my teaching. I have continued to use the same textbook[10] because I like it and it covers the material I need to teach. However, for one class period each week, we dispense with the textbook and have a hands-on manipulative session. For these sessions, I choose activities that complement the textbook materials.

Group Learning

During each manipulative session, we perform an experiment designed to uncover one or more mathematical facts. I divide the students into groups of two to four and encourage them to discuss their findings as they proceed. I walk around from group to group listening, observing, and asking questions to help them stay on task and reach a solution.

These sessions have several benefits. First, as students discuss their conjectures and findings, they learn to talk comfortably about mathematical ideas and terminology. They also learn to work together and respect other people's opinions. The slower student gets much more individualized instruction than in a lecture situation, where many students vie for the teacher's attention. The quicker students also benefit as they help their partners understand the concepts.[11]

Other positive outcomes from this type of activity have been noted by Lucille Croom of Hunter College of the City University of New York. She writes:

"Small-group, cooperative-learning experiences help students explore mathematical concepts in an interactive problem-solving setting. Research reveals that group interaction or cooperative learning promotes female and minority students' self-esteem, motivation, and achievement. Group interaction also promotes the development of mental operations or processes in children, since children tend to internalize the talk heard in the group. . . . Research shows that when students participate in cooperative learning, the attitudes toward their classmates, particularly those from different ethnic backgrounds, improve. . . . Students learn to respect other students' points of view and differences."[12]

While Croom refers especially to children, the ideas she expresses seem to work equally well for my students, who exclaim aloud and on paper how they wish they had learned math this way when they were in grade school. I wish I could demonstrate beyond reasonable doubt that learning that incorporates manipulatives actually is

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Manipulative sessions help the author's students to talk comfortably about mathematical ideas and terminology.

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superior to rote memorization. However, I am convinced that these methods have significantly reduced math phobia in my students.

Naturally, there is not enough time in one course to discover all of mathematics. Humans have been working on these discoveries for at least 3,000 years. So as teachers, we have to choose how to divide the material between discovery and lecture.

Types of Hands-on Activities

Hands-on mathematics experiments should be as varied as possible. Wolfram Eid of Otto-von-Guericke University in Germany insists that "pupils have to examine objects, to see connections to well-known things in mathematics."[13] Teachers can make their own hands-on learning materials or buy carefully crafted manipulatives from companies like Cuisenaire.

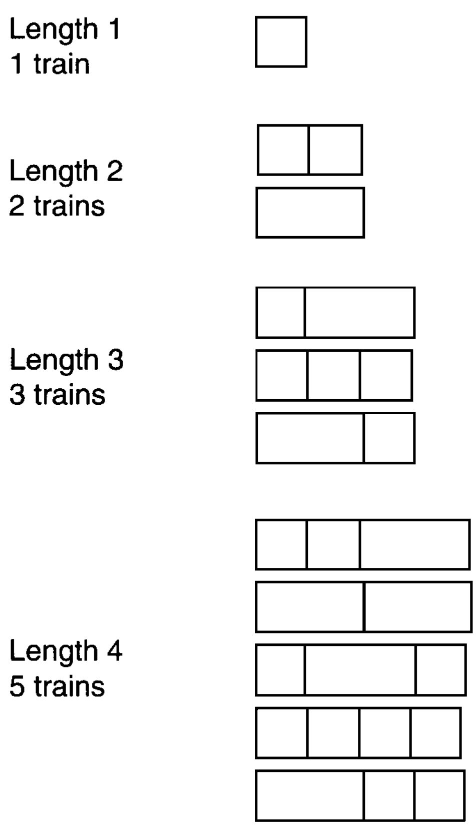
Rods

A favorite manipulative of mine is a set of Cuisenaire rods.[14] These unmarked, one-centimeter-square strips of wood or plastic come in lengths from 1 cm to 10 cm, with each length a different color. They were developed by Belgian educator Georges Cuisenaire and became quite popular during the 1960s. I encountered them first in Zimbabwe. If C-rods are not available, one can make substitutes from strips of one-centimeter-wide colored construction paper, dry spaghetti, or thatch grass cut to different lengths. These objects give students a physical feeling for the rules of addition, subtraction, multiplication, and division that they must memorize.

One of the first manipulative projects I try with a new class of prospective teachers is to ask them to make trains using rods of various lengths. I ask, "How many different trains can one make that are five centimeters long?" Each group elects one member to record the findings and report them to the class. Typically, each group figures out the pattern in about 10 minutes.

Next, I change the problem by restricting the types of rods to be used in constructing a train. Using one- and two-centimeter rods only, we get the trains illustrated in Figure 1.

Figure 1



Notice that to get Length 4 trains, all we need to do is attach Length 2 rod onto the right-hand end of Length 2 trains and a Length 1 rod onto the right-hand end of Length 3 trains. This technique works for each length beyond the first two. Table 1 summarizes the data for the first few trains.

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The students quickly see that to get the number of trains possible for any length, all they have to do is add the number of trains of the two immediately preceding lengths. They thus discover the Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, . . . named for the solution to the famous problem about rabbit reproduction that Fibonacci posed in his book *Liber Abacci*, published around 1202 A.D. Fibonacci introduced the Hindu-Arabic number system into Christian Europe. So this activity gives the teacher an opportunity to talk about the history of our numeration system.

An interesting and much harder variation is to allow no more than one rod of each length when constructing trains. Before dismissing this activity as "playing with blocks," read the article by the chair of the Department of Mathematics at Humboldt State University, "Cuisenaire Rods Go to College,"[15] in which she and her coauthors do a formal derivation of formulas for the number of trains that can be constructed under various criteria.

Finally, I have each student write a report on the activity. Writing about what they did forces students to think about mathematical processes and techniques and increases understanding.[16]

Technology in Mathematics

In addition to manipulatives, students should have a full range of mathematical tools available to them. The teacher must be skilled in using these tools, not as a substitute for, but as "power tools" to enhance student

Table 1

Length of Train	Number of Trains
1	1
2	2
3	3
4	5
5	8
6	13
7	21

problem-solving.

A logical place to start using technology is with electronic calculators, since most people either own or have access to these divices.[17] While there is still controversy about when children should be allowed to use a calculator, it is essential that they learn basic number facts. An activity like the one below can be used even before this point.

This beneficial activity for both language and mathematics predates calculators. It is based on the fact that, turned upside down, most numerals can be read as letters. I teach my students to start off by asking children a few questions such as:[18]

What do merchants do?
Calculate: 1547 x 5 =

What is the opposite of low?
Calculate: (159+192) x 14 =

Once children have caught on to the process, they can be directed to make up words using letters that can be represented by upside-down numerals. The next step is to have them make up math questions that yield those words as answers. As children work in groups, they will become excited over this challenge, and teachers will be amazed at their creativity.

For older students, graphing and symbolic manipulation calculators are becoming ever more popular. Again, there is a heated debate as to how much use should be made of them. Recently, the Advanced Placement exams in high school calculus have required their use. Allowing students to use graphing calculators during exams does require teachers to re-examine their testing techniques. Monique Boers of Swinburne University of Technology in Australia offers some insight into this problem:

"The graphics calculator provides students with graphical information that is independent of their algebraically derived information. Where the two do not agree, sometimes because of an error in the students' work or because of the inability of the student to relate the two, conflicts arise which have to be either resolved or ignored. Either way, the examiner gains a deeper insight into the students' level of understanding."[19]

Computers

Thousands of math programs are available, making the computer a very versatile tool. Some programs simply stress mastery of basic facts, while others use creative ways to help young people learn mathematics. Reading such journals as *Teaching Children Mathematics* published by the NCTM, attending conferences, and speaking with colleagues will help the teacher to make informed choices about which programs to use. Since the available math programs change almost daily, any published list would be obsolete. An excellent review of current.computer math programs can be found on the World Wide Web at: http://www.terc.edu/mathequity/gw/html/gwhome.html.[20] The University of Tennessee Knoxville produces advanced-level mathematics software available in the public domain (free) or as shareware (at a very nominal cost).[21]

When considering computer programs, don't overlook the utility of the simple spreadsheet, which can be put to a tremendous number of mathematical uses, even at the grade-school level. It quickly generates multiplication tables for any size numbers, will do graphs, find averages, and generate other useful statistics. Teachers also can use a spreadsheet for recording grades. Tony Steward of the University of Sunderland in England says that "Mathematical procedures are more easily and more visually represented on a spreadsheet than in a programming language."[22]

Several dynamic geometry programs allow students to explore concepts relating to distance, angles, areas, polygons, and circles. I have had considerable success with *Geometer's Sketchpad* from Key Curriculum Press.[23] I usually spend one of our manipulative sessions using a spreadsheet and one on *Geometer's Sketchpad.*

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Manipulatives give students a physical feeling for the rules of addition, subtraction, multiplication, and division that they must memorize.

Mathematics and History

Besides learning to use tools, my students also benefit from a brief overview of mathematics history. I

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spend between one and two weeks on this area. Students enjoy investigating the evolution of our Hindu-Arabic number system, which helps them understand why it works so well that it has been almost universally adopted.

Of course, a great reason to present math history is to humanize the subject. As students discover that mathematicians are real people with real lives, families, and problems, they can better relate to them. This helps them see mathematics as a human endeavor, rather than as a cold, impersonal discipline.

Ubiratan D'Ambrosio of the Universidad Estadual de Campinas in Sao Paulo, Brazil, describes mathematics as much "an integrating part of a culture, the same as language, arts, religions, and modes of explanation."[24]

Because I teach at a Christian university, I also spend a fair amount of class time on biblical numbers. Adventists would not exist if William Miller and his associates hadn't calculated the periods of prophecy. So combining mathematics with the study of the Bible is indeed an Adventist pastime. Our class explores why Christ chose 12 apostles rather than some other number and examines the significance of numbers such as 6, 7, 14, and 40 in the Bible.

Troublesome topics, like the implied biblical value of 3 for pi[25] and the number of people (50.070) God destroyed at Beth Shemesh[26] need to be discussed. My preliminary article "Numerology and the Hebrew Mind"[27] addresses some of these problems.

While my experiences do not prove that either traditional or reform methods of mathematics education work "better," I have found many new and interesting ways to make mathematics more meaningful and exciting for my students. This is a major step in alleviating math phobia and making mathematics accessible to a much larger percentage of future teachers.

NOTES AND REFERENCES

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3. See http://www.nctm.org/ for the *Principles and Standards for School Mathematics* document.

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5. George Malaty, "Eastern and Western Mathematical Education: Unity, Diversity, and Problems," *International Journal of Mathematics Education in Science and Technology* 29:3 (1998), p. 421.

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7. Matías Camacho, Martin M. Socas, and Josefa Hernandez, "An Analysis of Future Mathematics Teachers' Conceptions and Attitudes Towards Mathematics," *International Journal of Mathematics Education in Science and Technology* 29:3 (1998), p. 318.

8. See http://www.ncsu.edu/learn/saying.html/.

9. PROMPT (Professors Rethinking Options in Mathematics for Prospective Teachers). Check out the HSU Web site for details of many of the activities I have used: http://weasel.cnrs.humboldt.edu/~prompt/107/index.html.

10. See *The Nature of Mathematics* by Karl Smith (several editions) (Pacific Grove, Ca.: Brooks Cole).

11. Aaron Stucker, "Group Problem Solving," *Primus* (March 1992), p. 39. Stucker describes his experience in using small groups to solve calculus problems.

12. Lucille Croom, "Mathematics for All Students: Access, Excellence, and Equity," *Multicultural and Gender Equity in the Mathematics Classroom: The Gift of Diversity*, the 1997 Yearbook of the NCTM. Janet Trentacosta, ed. (Reston, Va.), p. 7.

13. Wolfram Eid, "Object-Oriented Modeling in Geometry," *International Journal of Mathematics Education in Science and Technology* 28:4 (1997) p. 473.

14. Marketed by Cuisenaire ETA/500 Greenview Court, Vernon Hills, IL 60061. A wealth of ideas on how to use the rods is given in their series of three books: *The Super Source: Cuisenaire Rods* published for grades K-2, 3-4, and 5-6, respectively. See also their Web site at http://www.etacuisenaire.com/index.htm/.

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17. Bert Waits, et al, "Calculators in Mathematics Teaching and Learning: Past, Present, and Future," in *Learning Mathematics for a New Century*, the 2000 Yearbook of the NCTM, Maurice Burke and Frances Curcio, eds. (Reston, Va.), pp. 51-66. The authors point out that in 1992, 81 percent of U.S. middle school students had calculators available, up from 21 percent in 1986, and discuss the implications for educators.

18. Karl Smith, *The Nature of Mathematics* (Pacific Grove, Calif.: Brooks/Cole, 1987), Smith has a nice collection of problems like these on pages 117 and 118.

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25. See 2 Chronicles 4:2, 3; and 1 Kings 7:23.

26. See 1 Samuel 6:19.

27. You can find this article on my Web site at http://www.lasierra.edu/~wclarke/. Follow the Math History link.

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