MANIPULATORS CAN MASTER Math

Anna (not her real name) had struggled with mathematical concepts since kindergarten. Each year, number sense, adding, subtracting, and multiplication seemed farther from Anna's grasp. But in the 5th grade, Anna's math teacher introduced her to the use of manipulatives. By using this new approach, Anna was able to solve equations such as $3x + 4 = 2x + 10$ with ease by the end of the third quarter.

The purpose of a classroom is to engage all learners in the process of education—even those who seem slow to comprehend certain concepts. Learning is a progressive experience, so teachers must firmly establish its foundation and provide opportunities for investigation, active involvement, and contemplation by each learner, which will facilitate the learning process. Educators Harry and Rosemary Wong estimate that in the typical classroom, students spend only 35 percent of the school day actively engaged in learning.[1] The Wongs recommend that teachers move students beyond listening to a lecture, answering textbook questions, and completing worksheets to become more actively engaged in the classroom.

The abstract nature of mathematics necessitates active participation by students in order to achieve mastery, especially when studying algebraic concepts. Since it is widely accepted that algebra is the gatekeeper course for college preparatory courses,[2] each math teacher needs to identify methods that will actively engage his or her students and create an environment where students can achieve success.

This article will provide a brief history of math manipulatives, review some of the research supporting their use in mathematics classrooms, and describe a few specific manipulatives that are available on the market today.

Burns and Humphreys suggest that gaps in the teaching of algebra prevent students from understanding why the subject is necessary for life. The typical algebra classroom focuses on "procedures, problems, rules, and rituals of algebra while failing to reveal its place in the world of mathematics."[3]

Definition and History of Manipulatives

Manipulatives can be any hands-on activities, interactive objects, or technology that teachers use to help students understand the objectives of the curriculum.[4]

In a historical study of manipulatives, Picciotto found that Zoltan Dienes was the first teacher on record to use manipulatives to help students comprehend algebraic concepts.[5] In 1963, Dienes assembled a team of educators, psychologists, and students at Harvard University in what became known as the Mathematics Learning Project. (At this point, the word *manipulatives* had not come into common usage.) Dienes studied the use of balance beams, blocks, hooks, and other items to represent math concepts. He found that these materials were bene-

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ficial in demonstrating a variety of mathematics principles such as the Distributive Law and Commutative Property but did not produce consistent results when used in factoring trinomials.[6] Mary Laycock enhanced the blocks used by Dienes by including multi-base blocks. Modifying both ideas, Peter Rasmussen worked with base ten tiles, which became the precursor to the popular Algebra Tiles and many other manipulatives.[7]

The International Center for Leadership in Education sponsors an annual Model Schools Conference to help schools develop academic programs that foster rigor and ensure relevance for all students. At these conferences, teachers and administrators from around North America present activities and programs that have helped their students experience success in every academic area. In the Keynote Address at the 2006 Model Schools Conference, Bill Daggett implored educators to present relevant assignments that move beyond the mere recall of facts to solving multifaceted real-world problems.[8] Many times, students who seem to know the facts of math fail to perform as expected on standardized tests. These students know that 12 x 12 = 144, but they don't know why. Until they do, math will simply be numbers in the textbook that have no connection to their lives.

Although some real-world problems may be included in the typical math textbook, Marzano and Pickering observed that application of new material outside of the textbook reinforces the significance of the information.[9] Moreover, math students in all grades need bridges to link the textbook with the real world.[10] Research suggests that manipulatives can provide that link, helping students recognize math applications outside of the textbook and classroom. (See Robert Moore's article on the use of manipulatives to measure a circle on page 30 of this issue.)

A 5th grader solves algebra equations with manipulatives.

Using Manipulatives Successfully

As stated above, manipulatives have been used to illustrate algebraic concepts for more than 40 years. Despite this fact, most pre-algebra and algebra students do not have regular contact with them. Perhaps their teachers don't see the need, haven't had training in how to align manipulatives with the textbook units, or aren't convinced that manipulatives can make a difference. In a two-year study of manipulatives in the algebra classroom, Leinenbach and Raymond compared the term averages before and after using manipulatives for one teacher's five algebra classes. When the students learned algebra only from a textbook, without using manipulatives, the average grade in the five classes was 78.89 percent. During and just after manipulatives use in teaching algebraic concepts, the mean score increased to 83.77 percent.[11]

In 1994, Ernest evaluated the training of 40 middle school and high school teachers on the use of manipulatives for algebra and geometry. The *Evaluation of Eisenhower Workshop* qualitative checklist was used to measure the teachers' success in using manipulatives after receiving training. She found that in most of the classrooms, 100 percent of the students participated when manipulatives were used, and the quality of instruction ranged from very good to excellent.[12]

Leinenbach and Raymond discovered that students' performance on assessment instruments improved when they actively participated in the learning process.[13] When evaluating the combined use of textbooks and manipulatives in mastering math objectives (contrasted with instruction using only the textbook), these researchers observed a 23 percent rise in students' test and quiz scores. Table 1 displays the difference in class averages using the textbook alone, and in combination with manipulatives.

Table 1.

Comparison of Class Averages Using Textbook and Manipulatives[14]

	Class Averages	
Class Period	Textbook	Manipulatives
1st	65%	82.38%
2nd	70.47%	81.28%
3rd	75.07%	85.29%
6th	81.29%	87.82%
7th	72.16%	82.1%

In a study of 7th and 8th graders in a multigrade mathematics classroom, the author of this article found that the students' attitudes about mathematics improved when manipulatives were combined with textbook use. In this study, the Mathematics Attitudes Survey (MAS) was given before and after students used manipulatives. The MAS measured student attitudes toward math in several areas. The results of the MAS revealed that when students used manipulatives, their attitudes toward math success, confidence while doing math, and usefulness of math were more positive than when manipulatives were not used. The study also found that students completed math assignments faster and with more accuracy when using manipulatives.[15]

These positive outcomes of combining manipulatives use with textbook instruction offer a stark contrast to what happens to students who experience constant failure in the math classroom. When Ewing interviewed 43 students regarding their high school math experience, he discovered that many had given up on math and consequently on school because they had become convinced that success was unattainable. The research subjects told Ewing that their teachers expected them to learn math from the textbook and seldom engaged them in active learning strategies. As a consequence, many of these students became bored and frustrated with assignments, failed exams, and eventually quit school. None of the students interviewed was in a classroom that used creative learning strategies such as manipulatives.[16]

Manipulatives Available

Moyer noticed that students in classrooms using manipulatives without stated objectives had more fun, but not higher math grades.[17] Unfortunately, teachers who are interested in using manipulatives that are aligned with objectives, textbooks, and state standards have few commercial alternatives. However, solutions can be found. Administrators should provide seminars and guidance in best practices for the use of manipulatives. After receiving training, math teachers can collaborate to harmonize manipulative use with curriculum objectives and experiment to determine which strategies work best with their students.

Many commercially produced manipulatives are suitable for pre-algebra, algebra, and of course, geometry classrooms. Because of space limitations, only a small sampling is listed in Table 2 on page 37. Some can be obtained as boxed sets; others must be ordered individually. Boxed sets tend to be more economical but may not contain sufficient pieces for an entire class. All of the manipulatives listed can be obtained from the Internet or ordered from mathematics catalogs.

The Challenge

In conclusion, research shows that when students use manipulatives,

- they are more successful in math classes;
- they participate more in the math classroom;
- their algebra quiz and test scores improve;
- their algebra term grades improve; and
- they have a more positive attitude toward mathematics.

Our students deserve every available opportunity to succeed. Math teachers need to create learning environments that will motivate students and prepare them for a global community. Manipulatives can help students to both ***do*** and ***understand*** mathematics.

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REFERENCES

1. Harry K. Wong and Rosemary T. Wong, *The First Days of School* (Mountain View, Calif.: Harry K. Wong Publications, 2005).
2. College Bulletin, "Students Learn by Doing Math," *Mathematics Department of East Carolina University* (Greenville, North Carolina). Retrieved January 31, 2013.
3. Marilyn Burns and Cathy Humphreys, *A Collection of Math Lessons From Grades 6 Through 8* (Sausalito, Calif.: Math Solutions Publications, 1998), p. 23.
4. Amanda Rust, "A Study of the Benefits of Math Manipulatives Versus Standard Curriculum in the Comprehension of Mathematical Concepts" (Master's Action Research Project, Johnson Bible College, Knoxville, Tenn., 1999), p. 3.
5. Henri Picciotto, "Algebra Manipulatives Comparison and History": http://www.picciotto.org/math-ed/manipulatives/alg-manip.html. Accessed January 31, 2013.
6. Zoltan Dienes, *An Experimental Study of Mathematics-Learning* (London: Hutchinson, 1963).
7. Picciotto, "Algebra Manipulatives Comparison and History," op. cit.
8. International Center for Leadership in Education, *The Education Challenge: Preparing Students for a Changing World* (Kissimmee, Fla.: 14th Model Schools Conference, June 2006).

Table 2. Selected Manipulatives for Algebra

MANIPULATIVE	DESCRIPTION/SUGGESTED USE	MARKET DESIGN
ALGEBRA TILES	These squares and rectangles help students visualize the distributive law and are beneficial in factoring polynomials.	
BASE TEN BLOCKS	An assortment of rods, cubes, and blocks that can be used to develop number sense, solve equations, and everything in between.	
GEOBOARDS	These square, usually 11- by 11-inch boards hold elastic in place on pegs as students experiment with perimeter, area, symmetry, fractions, angles, and various spatial activities.	
COLOR TILES	These versatile, 1-inch color tiles help develop basic arithmetic skills through the modeling of different math concepts.	
TANGRAMS	This set of seven geometric shapes can be used to support standards related to spatial sense by using the properties of parallelism, perpendicularity, and symmetry in solving real-world problems.	
HANDS-ON EQUATIONS	The fun and easy way to learn algebra for students in grades 3-8. Makes $4x + 3 = 3x + 9$ child's play!	
PATTERN BLOCKS	Multicolored, multi-shaped blocks that can be used to form amazing patterns.	
WRAP-UPS	Teachers can help students gain a strong foundation for upper-level math using these self-correcting Wrap-ups.	

9. Robert Marzano and Deborah Pickering, *Dimensions of Learning* (Alexandria, Va.: Association for Supervision and Curriculum Development, 1997).

10. Robert Reyes, et al., *Helping Children Learn Mathematics* (Englewood Cliffs, N.J.: Prentice Hall, 1989).

11. Marylin Leinenbach and Anne Raymond, "A Two-Year Collaborative Action Research Study on the Effects of a 'Hands-On' Approach to Learning." Paper presented at the annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Panama City, Fla., 1996.

12. Patricia Ernest, "Evaluation of the Effectiveness and Implementation of a Math Manipulatives Project." Paper presented at the annual meeting of the Mid-South Educational Research Association, Nashville, Tenn., November 1994.

13. Leinenbach and Raymond, "A Two-Year Collaborative-Active Study," op. cit.

14. Betty Nugent, "The Impact of Use of Mathematics Manipulatives on Attitudes Toward Mathematics Success Among Students in a Multigrade Classroom." Ed.D. dissertation, Walden University, Minneapolis, Minn., 2010.

15. Ibid.

16. Bronwyn Ewing, "'Go to the Page and Work It From There': Young People's Experiences of Learning Mathematics From a Text," *Australian Senior Mathematics Journal* 20:1 (2006):8-14.

17. Patricia Moyer, "Are We Having Fun Yet? How Teachers Use Manipulatives to Teach Mathematics," *Educational Studies in Mathematics* 47:2 (2001):175-197.